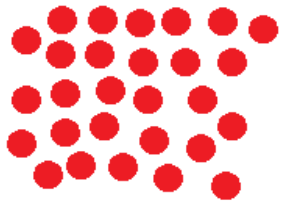
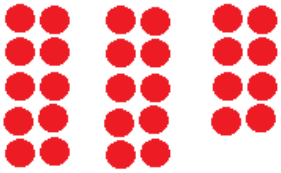
Number Systems

A number system is a way of grouping a value.

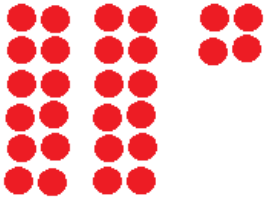
Instead of this:



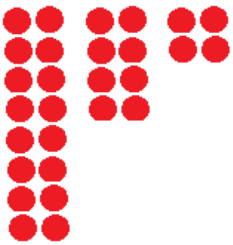
We can group the dots like this: 2 groups of 10, plus 8 or 28.



If they were eggs we might group them like this: 2 groups of 12, or 2 dozen, plus 4.



In the binary (base 2) number system we would group them like this: 1 group of 16, 1 group of 8, and 1 group of 4. (There are no groups of 2 or 1)



The number of dots in each picture is the same. They are just grouped differently.

The number system we use every day, the decimal number system, is based on 10.

Decimal Number System

The decimal number system is a place system. The right most place is the ones place. As we move from right to left, each place is 10 times the previous place.

Place:	1000	100	10	1
Digit:	1	0	4	3
Value:	1x1000	0x100	4x10	3x1

The number 1043 means 1 group of 1000, 0 groups of 100, 4 groups of 10 and 3 groups of 1.

You cannot leave out the 0 in the 100s place. 1043 is not the same as 143.

The place values can also be written as powers of 10. The two rows in the table below have the same values.

Place:	1000	100	10	1
Power:	10^3	10^2	10^1	10^0

In the decimal system, there are 10 digits 0, 1, ,2, 3, 4, 5, 6, 7, 8, 9. There can only be 1 digit in a place. You cannot put a 12 in the tens place. For 12 groups of 10, You would need to put a 2 in the 10s place, and a 1 in the hundreds place: 120. Sometimes we called that carrying: We carry a 1 to the hundreds place.

Binary Number System

The binary system uses a base of 2. There are 2 digits: 0 and 1. Binary is used in computers because each bit (**b**inary dig**it**) can have just one of two values. We usually represent those two values as 1 and 0, but it can also be thought of as true or false; on or off.

The binary number system is a place system. The right most place is the ones place. As we move from right to left, each place is 2 times the previous place.

Place:	8	4	2	1
Digit:	1	0	1	1
Value:	1x8	0x4	1x2	1x1

The value of the binary number 1011_2 =1x8 + 0x4 + 1x2 + 1x1 = 8 + 4 + 2 = 14_{10}.

For practical purposes we just add together the value of each place that has a one in it.

We read the binary number 1011 as "one zero one one." Do not say "one thousand eleven," those terms refer to the decimal system.

The place values can also be written as powers of 2. The two rows in the table below have the same values.

Place:	8	4	2	1
Power:	2^3	2^2	2^1	2^0

Convert Binary to Decimal

Let's convert the binary number 11011 to decimal. Start by writing the place values, starting on the right with one, then keep multiplying by 2 as you move to the right:

Place:	16	8	4	2	1
Digit:	1	1	0	1	1

Now add together the places that have a 1: 16+8+2+1= 27.

We can write this as $11011_2 = 27_{10}$ Say this as "one one zero one one in the base 2 equals twenty seven in the base ten." We often leave out "base 10" because that is the default number system for humans.

Let's do another one: Convert 11101_2 to decimal.

Place:	16	8	4	2	1
Digit:	1	1	1	0	1

Now add together the places that have a 1: 16+8+4+1= 29.

Now a hard one: 10111000_2

Place:	128	64	32	16	8	4	2	1
Digit:	1	0	1	1	1	0	0	0

Now add together the places that have a 1: 128+32+16+8= 184.

$10111000_2 = 184_{10}$

Convert Decimal to Binary

To convert the decimal number 236 to binary, write as many places as you need, starting on the right with the ones places and multiplying by 2 as you move to the left:

Place:	128	64	32	16	8	4	2	1

The next place would be 256, but we don't need it because 256 is more than 236, the number we are converting.

Next put a 1 in the right most place:

Place:	128	64	32	16	8	4	2	1
Digit:	1							

Now subtract 128 from 236: 236-128 = 108. In other words, we started with 236 and we used up 128 when we put a 1 in the 128 place, so we now have 108.

108 is more than 64, so we put a 1 in the 64 place and subtract 64.

Place:	128	64	32	16	8	4	2	1
Digit:	1	1						

Now subtract 64 from 108: 108-65 = 44.

44 is more than 32, so put a 1 in the 32 place and subtract 32.

Place:	128	64	32	16	8	4	2	1
Digit:	1	1	1					

Now subtract 32 from 44: 44-32 = 12.

12 is less than 16, so put a 0 in the 16 place.

Place:	128	64	32	16	8	4	2	1
Digit:	1	1	1	0				

12 is more than 8, so put a 1 in the 8 place.

Place:	128	64	32	16	8	4	2	1
Digit:	1	1	1	0	1			

12 is more than 8, so put a 1 in the 8 place and subtract 8 from 12. 12-8=4.

Place:	128	64	32	16	8	4	2	1
Digit:	1	1	1	0	1			

The next place is the 4's place and we have 4 left. We put a 1 in the 4's place and put a 0 in each of the remaining places:

Place:	128	64	32	16	8	4	2	1
Digit:	1	1	1	0	1	1	0	0

Check the answer: does 128+64+32+8+4 equal 236? Yes! Excellent!

Hexadecimal

If we write a large number as a binary number, it can be quite long.

The hexadecimal, (hex) base 16, is used because every 4 binary digits can be represented by one hexadecimal digit.

In binary, base 2, there are 2 digits: 0, 1.

In decimal, base 10, there are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

In hexadecimal, base 16, there are 16 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

The table below shows the corresponding values of decimal, binary, and hexadecimal:

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

To convert a binary number to hexadecimal, first break it into groups of 4 starting on the right. You can always add 0 on the left without changing the value of a number. If the left most group does not have 4 digits, just add zeroes on the left to make a group of 4. Then replace each group of 4 with the corresponding hexadecimal digit from the table above.

Example: Convert 1011011111011_2 to hexadecimal (base 16)

Break into groups of 4 starting on the RIGHT: 0001,0110,1111,1011

Write the hexadecimal digit for each group of 4:

0001,0110,1111,1011

1 6 F B

There we have it: $1011011111011_2 = 16FB_{16}$

The place values in hexadecimal start with the ones place on the right, then multiply by 16 for each place moving to the left:

1,048,576	**65,536**	**4096**	**256**	**16**	**1**
16^5	16^4	16^3	16^2	16^1	16^0

The hexadecimal number 16FB is shown below with the place values:

4096	**256**	**16**	**1**
1	6	F	B

The decimal value is 1x4096 + 6x256 + 15x16 + 11x1.

Or: 4096 + 1536 + 240 +11, which is 5883.

Fortunately, we don't usually have to convert hexadecimal to decimal.